RESEARCH MEMORANDUM

This is a working paper. It may be expanded, modified, or withdrawn at any time.

The RAND Corporation
SANTA MONICA • CALIFORNIA

SECRET

Prepared Under Contract AF 33(600)-36527

RESEARCH MEMORANDUM

A METHOD OF CONCEALING UNDERGROUND NUCLEAR EXPLOSIONS (U)

A.L. Latter, R.E. LeLevier, E.A. Martinelli and W. G. McMillan

RM-2347-AFT

March 30, 1959

Copy No. 7

Assigned to ______________________

The RAND Corporation
1700 MAIN ST. • SANTA MONICA • CALIFORNIA

SECRET

SUMMARY

It is shown theoretically that nuclear explosions can be effectively hidden in large underground cavities. An estimate of the effectiveness of the method indicates that a yield of more than 300 KT could be made to look seismically like a yield of 1 KT. Experiments with both chemical and nuclear explosions are needed to test the theory.

CONTENTS

INTRODUCTION*

The possibility of concealing nuclear explosions is of vital importance in connection with the Geneva Conference on test suspension. In this report we will show theoretically that the seismic signal from an underground explosion can be greatly reduced by exploding the bomb in a large cavity.

Last summer this idea was considered, but rejected for reasons which we now know to be fallacious. The argument went as follows. If a pressure p is suddenly applied in a cavity of radius a in an infinite elastic medium, the displacements at large distances are proportional(1) to pa^2. Since the

---

* Authors' Note: This report is essentially the content of the briefing presented by A. L. Latter to the Killian Committee at Livermore on 22 January 1959. The reader is also referred to the following reports:

a) H. A. Bethe, ''Theory of Seismic Coupling'' HAB-59-4

b) ''Report of the Panel on Seismic Improvement on the Concealment of Underground Explosions,'' 6 March 1959.

c) H. A. Bethe, ''Appendix to Theory of Seismic Coupling,'' HAB-59-5.

d) H. A. Bethe, ''Elaboration and Amendments to 'Concealment of Underground Explosions',\'' HAB-59-6.

(1) See, for example, K. E. Bullen, ''An Introduction to the Theory of Seismology,'' (Cambridge University Press, 1953).

energy W of the explosion is essentially proportional to pa^3, the displacement for a given energy is inversely proportional to a, and hence it appears that the coupling can be reduced by increasing the radius of the cavity. This reasoning, however, ignores an important physical effect. Because of preferential absorption of high frequencies in the earth, only the low frequency components of the wave are contained in the distant seismic signal. When this effect is taken into account, the displacement at great distances turns out to be proportional, as will be shown below, not to pa^2 but rather to pa^3, and therefore the distant signal is independent of the radius of the cavity. On this basis it was concluded last summer that a cavity would not be effective in decoupling the signal.

Actually the above argument is completely correct for holes which are big enough so that the pressure remains below the elastic limit of the surrounding medium. In other words, once the hole exceeds some critical size it does not pay to make it any bigger. The new discovery is that it does pay to have a hole which is big enough to eliminate any nonelastic behavior of the rock. Nonelastic behavior is bad because the medium can flow like a liquid and thus undergo large displacements.

ANALYSIS OF THE METHOD

Consider a nuclear explosion of yield W in a spherical cavity of radius a, sufficiently large that the pressure developed on the wall does not exceed the elastic limit of the medium. Assume that the energy W is suddenly distributed uniformly over the volume of the cavity producing a step-function pressure on the wall, given by

$$p = (\gamma - 1) \frac{W}{\frac{4}{3}\pi a^3}, \tag{1}$$

where γ is a constant characteristic of the gas in the hole. It is possible to achieve such a rapid and uniform distribution of the energy by use of radiation flow, as will be discussed below.

The Fourier transform of the elastic displacement produced by the step-pressure p, in the wave zone at a distance r, is[2]

$$\hat{\zeta}(\omega) = \frac{pa}{8\pi\mu r} \frac{c}{\omega_o^2 + i\omega_o\omega - \frac{\lambda+2\mu}{4\mu}\omega^2}, \tag{2}$$

where λ and μ are the Lamé constants, c is the sound speed, and

$$\omega_o = c/a. \tag{3}$$

We are interested in $\hat{\zeta}(\omega)$ for values of ω which are propagated to large distances with negligible attenuation in the earth. For the seismic network considered by the Geneva Conference the important range in ω is from 0 to about 6 sec^{-1}, corresponding to frequencies below 1 cps. For

(2) A. L. Latter, E. A. Martinelli and E. Teller, "A Seismic Scaling Law for Underground Explosions," RAND Paper No. P-1594, 14 January 1959, Eq. (4).

such low frequencies the first term in the denominator of Eq. (2) dominates in the cases of interest, and Eq. (2) becomes

$$\hat{\zeta}(\omega) = \frac{pa^3}{8\pi\mu rc} = \frac{3(\gamma-1)W}{32\pi^2\mu rc} . \qquad (4)$$

Thus we find, as has been pointed out by Latter, Martinelli and Teller,[2] that the amplitude of the distant seismic signal is proportional to pa^3, which is in turn proportional to W and independent of a. This is the basis of our earlier remark that the size of the hole does not affect the distant signal.

COMPARISON WITH RAINIER

It has been possible to give a simple analysis for the hole because the medium is assumed elastic. In the case of Rainier, however, we must deal with a shock wave in a medium whose equation of state is not known, and the transition of the shock wave to an elastic wave, which is not well-understood. For these reasons we shall not base our comparison on any theory of Rainier, but rather on direct experimental observations.

Measurements were made by Perret[3] of the accelerations produced by the Rainier Shot at distances of a few hundred feet from the explosion. The measurements made in the vertical direction at 371 and 451 feet are particularly useful since the medium was homogeneous and apparently elastic in this region, and the records were relatively free of extraneous reflections. The accelerometer records at these points have the form of a damped oscillatory wave characteristic of an elastic medium. We now apply Eq. (4) of reference (2) for the Fourier transform of the displacement at a distance r:

$$\hat{\zeta}(\omega) = \frac{\hat{p}a}{8\pi\mu}\left(\frac{1}{r^2} + \frac{i\omega}{rc}\right)\frac{c^2}{\omega_o^2 + i\omega_o\omega - \frac{\lambda+2\mu}{4\mu}\omega^2}, \qquad (5)$$

where a is the (unknown) radius at which the medium begins to behave elastically, $\hat{p}$ is the (unknown) Fourier transform of the pressure which acts at a, and $\omega_o = c/a$ as before.

(3) W. R. Perret, "Subsurface Motion from a confined Underground Detonation, Part I," Sandia Corp., Report No. ITR-1529, October 1957.

The Fourier transform of the acceleration is, of course, $-\omega^2\hat{\zeta}(\omega)$. We have tried to fit the experimental data by choosing various values for a and different forms for $\hat{p}$. We find that the value of a is about 280 feet, and therefore ω_o is about 25, essentially independent of the form of $\hat{p}$. More generally, for any yield W (in kilotons) exploded in the Rainier environment, the characteristic frequency ω_o must scale as $W^{-1/3}$ and hence

$$\omega_o = 25(1.7/W)^{1/3} \text{ sec}^{-1}, \qquad (6)$$

since the yield of Rainier was 1.7 kilotons.

From Eq. (5) and the experimental acceleration data we can now determine the Fourier transform of the displacement in the wave zone, which can then be compared directly with the same quantity for the hole given in Eq. (4). Denoting by r_o the radius at which the accelerations were measured (371 feet and 451 feet), and by r any radius in the wave zone, we find for $r \gg c/\omega \gg r_o$,

$$\hat{\zeta}(\omega) = \frac{r_o^2 i\omega}{rc} \hat{\zeta}_o(\omega), \qquad (7)$$

where $\hat{\zeta}_o$ is evaluated at r_o. The quantity $\hat{\zeta}_o$ is easily evaluated from the following expression:

$$\hat{\zeta}_o(\omega) = \frac{1}{2\pi i\omega} \int_0^\infty \dot{\zeta}_o(\tau) e^{-i\omega\tau} \, d\tau , \qquad (8)$$

where $\dot{\zeta}_o(\tau)$ is the velocity at r_o as a function of time. For the values of ω of interest, namely below about 6 sec^{-1}, we see from Eq. (6) that ω is less than ω_o provided $W < 100$ kilotons. In this case the exponential in

Eq. (8) is approximately unity for the important part of the integration $(0 < \tau < 1/\omega_o)$, and Eq. (8) reduces to

$$\hat{\zeta}_o(\omega) = \frac{d_o}{2\pi i\omega} , \tag{9}$$

where d_o is the displacement at r_o.

From Eqs. (7) and (9),

$$\hat{\zeta}(\omega) = \frac{r_o^2 d_o}{2\pi rc} . \tag{10}$$

This quantity is to be compared with the corresponding quantity for the hole given in Eq. (4). In what follows we affix the subscript h to all quantities referring to the hole. Taking the ratio of Eq. (10) to Eq. (4), we obtain

$$\frac{\hat{\zeta}(\omega)}{\hat{\zeta}_h(\omega)} = \frac{16\pi}{3(\gamma-1)} \frac{c_h}{c} \frac{\mu_h r_o^2 d_o}{W} . \tag{11}$$

For Rainier the quantity $r_o^2 d_o = 2 \times 10^9$ cm^3 or 2.4×10^9 cm^3 depending upon the measurement station. Choosing the smaller value, setting $W = 1.7$ KT $= 7 \times 10^{19}$ ergs, and letting $\gamma = 1.2$, which is appropriate for air under the conditions of interest, we find for the decoupling factor

$$\frac{\hat{\zeta}(\omega)}{\hat{\zeta}_h(\omega)} = 2.4 \frac{c_h}{c} \mu_h, \tag{12}$$

where μ_h is in kilobars. If the hole were in the Rainier medium, $c_h = c$ and $\mu_h \sim 20$ kb, and the decoupling factor is ~ 50. Obviously it is desirable to make the hole in a stronger medium, e.g., salt, for which $c_h \sim 2.5$ c and $\mu_h \sim 100$ kb. The decoupling factor in this case is ~ 600. This number should be reduced somewhat because the strong medium will couple eventually to softer surroundings, and an overshoot will occur. This overshoot can be at most a factor of 2, and therefore we have a decoupling factor of at least 300.

Another comparison with Rainier can be made on the basis of the USC and GS seismic measurements at a distance of about a kilometer. These measurements showed that the energy which got into the seismic field outside a radius of a few hundred feet was approximately five per cent of the total yield.(4)

The total seismic energy radiated by the hole is

$$E_h = \frac{\pi}{2} \frac{p^2 a^3}{\mu}, \qquad (13)$$

where, as before, p is the step-pressure in the hole of radius a, and is related to the total yield W through Eq. (1). In order to ensure that the frequency analysis of the seismic wave produced by the hole be comparable to that of the Rainier shot, we must choose $a \approx 280$ ft. Again setting $\mu = 20$ kb, we find that $E_h = 2 \times 10^{-5} W$. Hence the seismic energy from Rainier was about $\frac{5 \times 10^{-2}}{2 \times 10^{-5}} = 2500$ times as great as that expected from the hole. Since the displacement is proportional to the square root of the energy E_h it follows that the decoupling factor is $\sqrt{2500} = 50$. This figure is in excellent agreement with our previous estimate for a hole in the Rainier medium.

(4) D. S. Carder, W. K. Cloud, L. M. Murphy, J. H. Hershberger, "Surface Motions from an Underground Explosion", U.S.C. and G.S. Report WT 1530, Nov. 14, 1958.

SIZE AND SHAPE OF THE HOLE

We have seen theoretically that the seismic signal produced by an explosion in a hole in a strong medium will be hundreds of times smaller than that from the same yield under Rainier conditions (in Nevada tuff without a hole). To what extent such a large decoupling factor can be realized in practice depends mainly on being able to obtain a big hole in the right kind of medium.

The size of the hole which is required, is proportional to the yield of the explosion and inversely proportional to the pressure to which the surrounding medium can be subjected without behaving inelastically. Both plasticity and cracking must be avoided. The latter may impose a severe limit since most rocks possess little strength in tension. Precisely what the limiting pressure is, must be determined from experiment, but in the meantime, it seems safe to say that the elastic requirement will be fulfilled if the explosion pressure does not greatly exceed the overburden pressure. The reason is that the overburden produces a compressive stress which can be relieved by the explosion without putting the medium into tension.

Considering the depth at which cavities can be easily ''washed out'' in salt domes (which seem to be a particularly suitable medium), an explosion pressure of about 150 atmospheres seems feasible—though, of course, we hope experiment will prove that the value can be considerably increased. Using 150 atmospheres, it is easily calculated from Eq. (1)

that the volume of the hole must be 3×10^6 ft^3 (radius 90 ft) for a yield of 1.7 KT, where we have set $\gamma = 1.2$, appropriate to air under the assumed conditions. In general the volume of the hole must be approximately

$$V = 3 \times 10^5 \frac{W_{KT}}{p_{kb}} ft^3, \qquad (14)$$

where W_{KT} is the yield in kilotons, and p_{kb} is the maximum permissible pressure in kilobars.

It may be necessary for practical reasons to use a nonspherical hole. If so, the ratio of the major to the minor radius must not be so large that the explosion energy is effectively trapped in less than the total volume of the cavity. This problem has not been studied in detail, but it seems clear that a ratio of three or four is quite safe.

Assuming that pressure spikes on the wall of a nonspherical cavity can be avoided, there is still the question of the nature of the seismic radiation produced, how much and what frequencies. This question too remains to be studied. A crude analysis suggests that for a class of prolate spheroids of fixed volume, the total seismic energy will not vary significantly. Furthermore the frequency spectrum may be shifted somewhat toward higher values corresponding to the minor dimension of the spheroid. If the spectrum is shifted upward, and if the total seismic energy is the same, then making the hole nonspherical will further reduce the distant seismic signal.

NATURE OF THE PRESSURE PULSE

In air at density ρ, the shock wave from an explosion of W kilotons forms at a radius given by

$$R = 11.5\, W^{0.3}(\rho_o/\rho)^{0.5} \text{ft}, \qquad (15)$$

where ρ_o is the density of normal sea-level air. If the radius of the hole is larger than R, the air will shock up and the pressure experienced by the wall will rise for a short time considerably above the average pressure in the hole. Provided such a pressure spike does not cause the surrounding medium to behave inelastically, it will not increase the distant seismic signal because the spike lasts for a time which is very short compared to the periods of the waves that can propagate to large distances. As an example, consider an explosion of 1.7 KT in a spherical hole of radius 130 ft. If the hole contains air at normal density, the wall will experience a pressure spike of about a kilobar acting for a few milliseconds. Thereafter the pressure on the wall will be sensibly the same as the average pressure in the hole, namely about 50 atmospheres.

We do not know whether a kilobar acting for a few milliseconds will cause any important nonelasticity. Such a pressure may well be below the plastic limit and of too short duration to produce large tensile hoop stresses. However, if it turns out to be necessary to reduce the pressure spike or eliminate it entirely, this appears to be easy to do by partially evacuating the cavity or by partially replacing the air with hydrogen gas. Either of these measures will increase the radiation phase of the fireball growth, allowing less energy to go into shock motion. If, for example, the air density in the cavity were reduced to 0.01 normal, the radiation phase would extend to the full 130 foot radius in the case we have just considered. In this way the pressure on the wall could be made approximately a step-pressure of 50 atmospheres, with no spikes.

In addition to the possible pressure spike from the shock wave, the hot gas in the cavity will interact with the wall and cause some vaporization ("blow-off") of the surface. This effect will produce a recoil shock pressure which must be added to the static pressure of the gas. The magnitude of the shock depends on the rate of vaporization and hence sensitively on the temperature of the hot gas. For the case of 1.7 KT in a 130 foot hole containing normal density air, the average temperature is only $\sim$1 e.v., and the blow-off pressure is utterly negligible. If, however, the hole is evacuated to 0.01 normal density to eliminate the shock, the temperature is $\sim$12 e.v. and it is not quite certain that the blow-off pressure is still negligible, though such rough calculations as have been made do come out that way. If the blow-off pressure should prove to be appreciable, it can be eliminated simply by introducing additional material in the hole to soak up energy and lower the temperature. This additional material should be in the form of foils which are thin to absorb energy rapidly and oriented along radial lines so as not to impede the outward flow of radiation.

There is a possible further advantage of the foils because some of the energy they absorb goes into nonpressure forms such as latent heats of melting and vaporization. While it does not seem likely that the pressure could be greatly reduced in this way in a time which is short compared to the period of the important seismic waves, a factor of 2 or 3 may well be possible. This point needs further investigation.